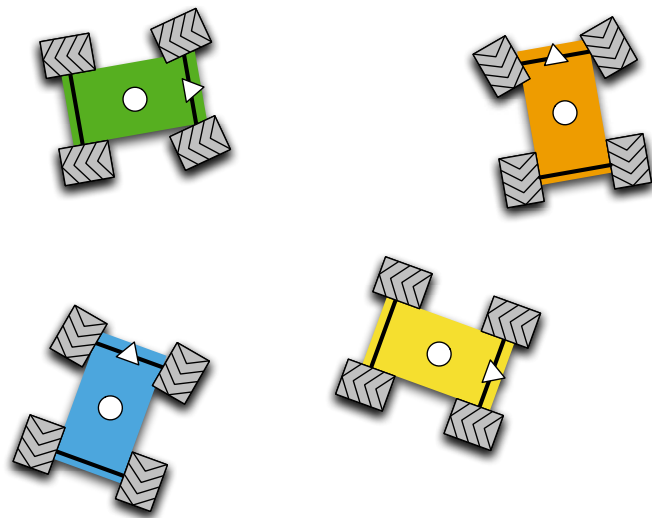

Distributed Control of Robotic Networks

A Mathematical Approach to Motion Coordination Algorithms

Francesco Bullo

Jorge Cortés

Sonia Martínez



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Symbol Index

γ_{arc}	: arc-length parametrization, 7
$O(g)$: big O Bachmann–Landau symbol, 5
$\Omega(g)$: big Omega Bachmann–Landau symbol, 5
$\Theta(g)$: big Theta Bachmann–Landau symbol, 5
∂S	: boundary of the set S , 3
$ S $: number of elements of the finite set S , 3
$S_1 \times S_2$: Cartesian product of S_1 and S_2 , 4
$\prod_{a \in A} S_a$: Cartesian product of the collection of sets $\{S_a\}_{a \in A}$, 4
S^n	: Cartesian product of n copies of S , 4
S_δ	: δ -contraction of S , 97
δ^S	: symmetric difference, 252
$\phi : \mathbb{R}^d \rightarrow \mathbb{R}_{\geq 0}$: density function on \mathbb{R}^d , 101
\emptyset	: the empty set, 3
$G \cap G'$: intersection of graphs G and G' , 23
$G \cup G'$: union of graphs G and G' , 23
$\mathbb{G}(S)$: set of all undirected graphs whose vertex set is an element of $\mathbb{F}(S)$, 104
$H_{p,q}$: closed halfspace defined by p and q , 96
$H_S(v)$: internal tangent halfplane of v with respect to S , 98
$[a, b]$: closed interval between the numbers a and b , 4
$]a, b[$: open interval between the numbers a and b , 4
$f : S \rightarrow T$: map f from set S to set T , 4
$f \circ g$: composition of the maps f and g , 4
f^{-1}	: inverse map of a function f , 4
$f^{-1}(x)$: level set of a function f corresponding to a value x , 4
T_f	: overapproximation map associated to a time-dependent evolution f , 22
$h : S \rightrightarrows T$: set-valued map h from set S to set T , 4
$A > 0$: a symmetric positive definite matrix A , 8
$A \geq 0$: a symmetric positive semidefinite matrix A , 8

A^T	: transpose of a real matrix A , 8
U^*	: conjugate transpose of a complex matrix U , 8
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\mathcal{S}_F	: network associated to $F \in \mathbb{R}^{n \times n}$, 55
$\mathcal{S}_{\text{disk}}$: network of first-order robots with range-limited communication, 142
\mathcal{S}_{D}	: network of first-order robots with Delaunay communication, 143
\mathcal{S}_{LD}	: network of first-order robots with range-limited Delaunay communication, 143
$\mathcal{S}_{\infty\text{-disk}}$: network of first-order robots with r - ∞ -disk communication, 143
$\mathcal{S}_{\text{vehicles}}$: network of planar vehicle robots with Delaunay communication, 143
$\mathcal{S}_{\text{vis-disk}}$: network of robots with line-of-sight communication, 143
$\mathcal{S}_{\text{circle}}$: network of first-order robots in \mathbb{S}^1 , 144
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$\mathcal{S}_{\text{vis-disk}}^{\text{rs}}$: network of robots with line-of-sight relative sensing, 155
$\ x\ _p$: L^p -norm of a vector x , 5
$\ A\ _p$: p -induced norm of a matrix A , 11
\mathcal{P}_I	: s -partition of I , 163
$f_-(a)$: limit from the left of f at a , 116
$f_+(a)$: limit from the right of f at a , 116
$\mathbf{J}_{\phi}(S, p)$: polar moment of inertia of S about p with respect to ϕ , 101
$w^{[i]}$: state of processor i , 40

- $w_0^{[i]}$: initial state of processor i , 41
 $W^{[i]}$: state set of processor i , 40
 $W_0^{[i]}$: set of allowable initial values for processor i , 40
 $r_{\text{exp}}(A)$: exponential convergence factor of $A \in \mathbb{R}^{n \times n}$, 61
 Σ^{b} : body reference frame, 151
 Σ^{fixed} : fixed reference frame, 151
 $[p, q]$: closed segment with extreme points p and q , 96
 $]p, q[$: open segment with extreme points p and q , 96
 $\text{rbt-sns} : \mathbb{R}^d \rightarrow \mathbb{A}_{\text{rbt}}$:
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 $\text{env-sns} : \mathbb{P}(\mathbb{R}^d) \rightarrow \mathbb{A}_{\text{env}}$:
 environment sensing function, 153
 $\{S_a\}_{a \in A}$: collection of sets indexed by the index set A , 4
 $x \in S$: x is an element of the set S , 3
 $R \subset S$: R is a subset of S , 3
 $R \subsetneq S$: R is a strict subset of S , 3
 $S_1 \cap S_2$: intersection of sets S_1 and S_2 , 4
 $\bigcap_{a \in A} S_a$: intersection product of the collection of sets $\{S_a\}_{a \in A}$, 4
 $S_1 \cup S_2$: union of sets S_1 and S_2 , 4
 $\bigcup_{a \in A} S_a$: union of the collection of sets $\{S_a\}_{a \in A}$, 4
 e_i : the vector in \mathbb{R}^d whose entries are zero except for the i th entry, which is one, 4
 $\mathbf{1}_d$: the vector in \mathbb{R}^d whose entries are all equal to one, 4
 $\mathbf{1}_{d-}$: shorthand for $(1, -1, 1, \dots, (-1)^{d-2}, (-1)^{d-1}) \in \mathbb{R}^d$, 67
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 \mathbb{A}_{env} : environment sensing alphabet, 153
 $A_\phi(S)$: area of S with respect to ϕ , 101
 $\text{avrg}(S)$: average of points in S , 181
 $A(G)$: adjacency matrix of G , 27
 $\text{ATrid}_n^\pm(a, b)$: augmented tridiagonal matrix, 66
 $B(x, \varepsilon)$: open ball of center x and radius ε , 6
 $\overline{B}(x, \varepsilon)$: closed ball of center x and radius ε , 6
 $\text{CM}_\phi(S)$: centroid or center of mass of S with respect to ϕ , 101
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 \mathcal{CC} : control and communication law, 145
 $\mathcal{CC}_{(s, \mathcal{P}_I)}$: \mathcal{P}_I -rescheduling of \mathcal{CC} , 164
 ctl : motion control function, 145
 $\text{co}(S)$: convex hull of S , 96
 κ_{abs} : absolute curvature, 8
 κ_{signed} : signed curvature, 8
 \mathbb{C} : set of complex numbers, 4
 $\mathbb{C}^{n \times m}$: set of $n \times m$ complex matrices, 8
 $\text{Circ}_n(a, b, c)$: tridiagonal circulant matrix, 64
 $D_{\text{in}}(G)$: weighted in-degree matrix of G , 28
 $D_{\text{out}}(G)$: weighted out-degree matrix of G , 28
 $d_{\text{in}}(v)$: weighted in-degree of a vertex v , 28
 $d_{\text{out}}(v)$: weighted out-degree of a vertex v , 28
 $\text{diag}(S^n)$: diagonal set of the Cartesian product S^n , 4
 $\text{diag}(v)$: square matrix with components of vector v in the diagonal, 8
 $\text{diam}(S)$: diameter of the set S , 7
 $\text{diam}(G)$: diameter of G , 28
 $\text{Dscn}(f)$: set of points where f is discontinuous, 116
 dist : distance function, 5, 6
 $\mathcal{D}_{\text{curvature}}(q_i, q_j)$: curvature distance between q_i and q_j , 254
 $\mathcal{D}_{\text{arc}}(q_i, q_j)$: arc-length distance between q_i and q_j , 254
 dist_{c} : clockwise distance, 5
 dist_{cc} : counterclockwise distance, 5
 dist_g : geodesic distance, 5
 dist_p : L^p -distance, 5
 $\mathcal{D}_\lambda(q_i, q_j)$: pseudo-distance between q_i and q_j , 255
 dist_G : distance in G , 28
 wdist_G : weighted distance in G , 29
 \mathcal{DA} : distributed algorithm, 40
 $\mathcal{E}_{\mathcal{G}}$: edge map associated to \mathcal{G} , 104
 $E(G)$: edges of G , 22

E_{cmm}	: set of communication links in a network of processors or in a robotic network, 39
$\text{Ed}(Q)$: edges of Q , 97
\mathcal{ECC}	: event-driven control and communication law, 248
$\text{Fa}(Q)$: faces of Q , 97
$\mathbb{F}(S)$: collection of finite subsets of the set S , 3
fti	: from-to-inside function, 96
G	: a graph or a digraph, 22
\mathcal{G}	: proximity graph, 104
\mathcal{G}_D	: Delaunay graph, 105
$\mathcal{G}_{\text{EMST}, \mathcal{G}}$: Euclidean minimum spanning tree of \mathcal{G} , 106
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\mathcal{G}_G	: Gabriel graph, 106
$\mathcal{G}_{\text{lc}, \mathcal{G}}$: locally cliqueless graph of \mathcal{G} , 110
\mathcal{G}_{RN}	: relative neighborhood graph, 105
$\mathcal{G}_{\text{cmlpt}}$: complete proximity graph, 104
$\mathcal{G}_{\text{disk}}(r)$: r -disk graph, 105
$\mathcal{G}_{\infty\text{-disk}}(r)$: r - ∞ -disk graph, 106
$\mathcal{G}_{\text{LD}}(r)$: r -limited Delaunay graph, 105
$\mathcal{G}_{\text{vis}, Q}$: visibility graph in Q , 106
$\mathcal{G}_{\text{vis-disk}, Q}$: range-limited visibility graph in Q , 106
id_S	: identity map on a set S , 4
I_n	: $n \times n$ identity matrix, 8
$\text{image}(f)$: image of the map f , 4
$i_{\mathbb{F}} : X^n \rightarrow \mathbb{F}(X)$: natural immersion of X^n into $\mathbb{F}(X)$, 110
$\text{IC}(S)$: incenter of S , 100
1_R	: indicator map associated with a set R , 4
$\text{IR}(S)$: inradius of S , 100
$\text{int}(S)$: interior of the set S , 3
I	: set of unique identifiers, 39
$\text{kernel}(S)$: visibility kernel set of S , 97
$\text{kernel}(A)$: kernel subspace of a matrix A , 8
$L(G)$: Laplacian matrix of G , 38
$\mathcal{L}_f V$: Lie derivative of a function V along a vector field f , 18
msg-gen	: message-generation function, 248
msg-rec	: message-reception function, 248
msg-trig	: message-trigger function, 248
msg	: message-generation function, 40

$\mathcal{M}(x, w)$: set of all non-null messages generated during one communication round from (x, w) , 161
MST	: minimum-weight spanning tree, 34
$\mathcal{N}_G(v)$: set of neighbors of v in G , 23
$\mathcal{N}_G^{\text{in}}(v)$: set of in-neighbors of v in G , 23
$\mathcal{N}_G^{\text{out}}(v)$: set of out-neighbors of v in G , 23
\mathcal{N}_G	: set of neighbors map of G , 107
null	: null message, 40
\mathbb{N}	: set of natural numbers, 4
\mathbf{n}_{out}	: outward normal vector, 8
$\text{PCC}(S)$: parallel circumcenter of S , 196
$\mathbb{P}(S)$: collection of subsets of the set S , 3
proj_W	: projection onto the set W , 7
$\mathbb{R}^{n \times m}$: set of $n \times m$ real matrices, 8
\mathbb{R}	: set of real numbers, 4
$\mathbb{R}_{\geq 0}$: set of non-negative real numbers, 4
$\mathbb{R}_{> 0}$: set of positive real numbers, 4
$\text{radius}(v, G)$: radius of v in G , 28
ρ	: radius of curvature, 8
$\text{rank}(A)$: rank of a matrix A , 8
$\text{rco}(S; X)$: relative convex hull of S in X , 99
\mathcal{R}	: set of mobile robots, 141
$\text{SC}(\mathcal{DA})$: space complexity of a distributed algorithm \mathcal{DA} , 43
$\text{SC}(\mathcal{T}, \mathcal{CC})$: space complexity to achieve \mathcal{T} with \mathcal{CC} , 161
$\rho(A)$: spectral radius of a matrix A , 11
$\rho_{\text{ess}}(A)$: essential spectral radius of a matrix A , 12
$\text{spec}(A)$: spectrum of a matrix A , 10
\mathbb{S}^d	: sphere of dimension d , 4
stf	: state-transition function, 40
stf-trig	: state-transition trigger function, 249
$T_\varepsilon(A)$: ε -convergence time of $A \in \mathbb{R}^{n \times n}$, 61
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$T\mathbb{S}^d$: tangent space of \mathbb{S}^d , 4
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$\mathcal{T}_{\varepsilon\text{-eqdstnc}}$: agent equidistance task, 256
\mathcal{T}_{dir}	: direction agreement task, 159
$\mathcal{T}_{\varepsilon\text{-eqdstnc}}$: equidistance task, 159

- $\mathcal{T}_{\text{rndzvs}}$: rendezvous task, 181
 $\mathcal{T}_{\varepsilon\text{-rndzvs}}$: ε -rendezvous task, 181
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 $\mathcal{T}_{\varepsilon\text{-}r\text{-area-dply}}$: ε - r -area deployment task, 221
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 $\mathcal{T}_{\varepsilon\text{-dc-dply}}$: ε -disk-covering deployment task, 222
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 $\text{TC}(\mathcal{T}, \mathcal{CC}, x_0, w_0)$:
 time complexity to achieve \mathcal{T} with \mathcal{CC} from (x_0, w_0) , 160
 TSP : traveling salesperson tour, 34
 T_{BFS} : breadth-first spanning (BFS) tree, 29
 T_{DFS} : depth-first spanning (DFS) tree, 31
 $T_{\text{shortest-paths}}$: shortest-paths tree, 32
 $\text{Trid}_n(a, b, c)$: tridiagonal Toeplitz matrix, 64
 $\text{vers} : \mathbb{R}^d \rightarrow \mathbb{R}^d$: versor operator, 240
 $V(G)$: vertices of G , 22
 $\text{Ve}(Q)$: vertices of Q , 97
 $\text{Vi}(p; S)$: set of all points in S visible from p , 97
 $\text{Vi}_{\text{disk}}(p; S)$: set of all points in S within a distance r and visible from p , 97
 $V_i(\mathcal{P})$: Voronoi cell of p_i , 102
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 $\mathcal{V}_r(\mathcal{P})$: r -limited Voronoi partition generated by $\mathcal{P} = \{p_1, \dots, p_n\}$, 102
 $\mathbb{Z}_{\geq 0}$: set of non-negative integer numbers, 4